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Measuring the Strangeness Radius of the Proton

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Parity violation in elastic $\vec{e}p$ scattering determines the elastic weak neutral current form factors $G_E^Z(q^2)$ and $G_M^Z(q^2)$ which are directly analogous to the familiar electromagnetic elastic form factors. By using the Standard Model of electroweak interactions and invoking strong isospin for the nucleon, the weak neutral current form factors may be expressed in terms of their electromagnetic counterparts and the strange quark vector form factors $G_E^{(S)}(q^2)$ and $G_M^{(S)}(q^2)$. Previous authors have argued that it is feasible to measure $G_M^{(S)}(q^2=0)$ in this way. In this paper, we show one may also measure $G_E^{(S)}(q^2)$ at finite q^2 in a separate experiment. Within certain models, it should be possible to extract the "strangeness radius" of the proton from such data.

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The discovery by the EMC collaboration[1] that the "Ellis-Jaffe" sum rule[2,3] is violated has led to the supposition that the strange quark matrix element $< p|\bar{s}\gamma_\mu\gamma_5 s|p >$ is large.[4] In addition, there has long been the suggestion that the scalar matrix element $< p|\bar{s}s|p >$ is also large.[4,5] Consequently, it is natural to wonder if matrix elements of the vector operator $\bar{s}\gamma_\mu s$ are also nonzero.[6] McKeown[7], Beck[8], and Decker and Lieze[9] have shown that parity violation in elastic $\vec{e}p$ scattering is sensitive to such vector matrix elements. The basic idea is that since parity violation measures the Z^0 coupling to the proton, it is sensitive to a different linear combination of the u, d, and s currents. One then invokes isospin to isolate the strange current matrix elements using the known electromagnetic properties of the proton and neutron. In fact, such an experiment is currently underway[10] to measure the $SU(3)$-singlet anomalous vector form factor $F_2^{(0)}$ near $q^2 = 0$, from which one can extract the analogous strange form factor $F_2^{(S)}$. In this paper we point out that in a separate experiment one may also determine the form factor $F_1^{(S)}$ over a finite q^2 range. We illustrate the sensitivity of such an experiment using relations for $F_1^{(S)}(q^2)$ and $F_2^{(S)}(q^2)$ derived by Jaffe.[11] In these relations the $q^2 = 0$ values of the strange form factors are typified by the strange quark contribution to the proton magnetic moment (for $F_2^{(S)}(0)$) and the strangeness radius of the proton (for $F_1^{(S)}(0)$).

Within the framework of the Standard Model, it is straightforward to determine an expression for the parity violating asymmetry in elastic $\vec{e}p$ scattering. One starts by writing the matrix elements for a vector or axial vector operator, V_μ or A_μ respectively, in terms of the relevant form factors $F_1(q^2)$ and $F_2(q^2)$ or $G_A(q^2)$ respectively, i.e.

$$< p'|V_\mu|p > = \bar{\mathcal{U}}\left[F_1(q^2)\gamma_\mu + \frac{i}{2M}F_2(q^2)\sigma_{\mu\nu}q^\nu\right]\mathcal{U}$$

$$< p'|A_\mu|p > = \bar{\mathcal{U}}G_A(q^2)\gamma_\mu\gamma_5\mathcal{U}$$

The parity violating asymmetry is then given by[7,8,12,13]

$$
\begin{aligned}
A &\equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \\
&= -\frac{G_F Q^2}{\pi\alpha\sqrt{2}} \Big[\varepsilon\, G_E^{\gamma} G_E^{Z} + \tau\, G_M^{\gamma} G_M^{Z} \\
&\qquad\qquad -\frac{1}{2}(1-4sin^2\theta_W)(1-\varepsilon^2)^{1/2}\sqrt{\tau(1+\tau)}G_M^{\gamma} G_A^{Z}\Big] \\
&\quad \times \left[\varepsilon\,(G_E^{\gamma})^2 + \tau\,(G_M^{\gamma})^2\right]^{-1} \qquad (1)
\end{aligned}
$$

where we make use of the "electric" and "magnetic" form factors $G_E \equiv F_1 - \tau F_2$ and $G_M \equiv F_1 + F_2$, with $\tau \equiv Q^2/4M^2$ and $Q^2 \equiv -q^2 > 0$. Note that for a fixed q^2, the parameter $\varepsilon \equiv \left[1 + 2(1+\tau)tan^2(\theta/2)\right]^{-1}$, where θ is the electron scattering angle, controls sensitivity to the "electric", "magnetic", or "axial vector" terms. Note also that in our normalization, one has (in the absence of a strange axial vector contribution) $G_A^Z(0) = \frac{1}{2}G_A^W(0) \approx \frac{1}{2}(-1.262)$. Radiative corrections to the asymmetry have not been included, but they seem to be well in hand.[14]

The form factors represent our ignorance about the nonperturbative structure of the proton, although the electromagnetic form factors $G_E^{\gamma}(q^2)$ and $G_M^{\gamma}(q^2)$ are well known from differential cross section measurements. The weak neutral form factors $G_E^Z(q^2)$ and $G_M^Z(q^2)$ (as well as $G_A^Z(q^2)$) are not known and would be measured in a parity violation experiment. However, the Standard Model relates the weak neutral and electromagnetic currents via $J_\mu^Z = J_\mu^{Weak\ Isospin} - \sin^2\theta_W J_\mu^{\gamma}$ which helps us relate the corresponding form factors. If one writes the vector parts of these currents in terms of the quark currents $\bar{q}\gamma_\mu q$ and relates the neutron and proton form factors using strong isospin[8,12,13], we find

$$
\begin{aligned}
G_E^Z(q^2) &= \left(\frac{1}{4} - sin^2\theta_W\right) G_{E_p}^{\gamma}(q^2) - \frac{1}{4}G_{E_n}^{\gamma}(q^2) - \frac{1}{4}G_E^{(S)}(q^2) \\
G_M^Z(q^2) &= \left(\frac{1}{4} - sin^2\theta_W\right) G_{M_p}^{\gamma}(q^2) - \frac{1}{4}G_{M_n}^{\gamma}(q^2) - \frac{1}{4}G_M^{(S)}(q^2) \qquad (2)
\end{aligned}
$$

Here we explicitly write the subscripts "p" and "n" to indicate proton or neutron electromagnetic form factors. (We note, however, that the assumption of isospin at finite q^2 has been questioned for precision analyses.[13]) The form factors $G^{(S)}_{E,M}(q^2)$ are derived from the current $\bar{s}\gamma_\mu s$. From here on, we suppress the superscript "γ" on the electromagnetic form factors.

The current experimental effort[10] aims to measure $G^Z_M(q^2 \approx 0)$. Since strangeness cannot contribute to the nucleon charge, we know that $G^{(S)}_E(0) = F^{(S)}_1(0) = 0$ and so one can deduce the value of $G^{(S)}_M(0)$ or, equivalently, $F^{(S)}_2(0)$. The experiment takes data at $\varepsilon \approx 0$, i.e. backward scattering angles, so that the "electric" contribution to Eqn. 1 is suppressed. One also works at a low enough Q^2 so that the extrapolation to $Q^2 = 0$ is reliable, but high enough Q^2 so that the asymmetry A is relatively large. This experiment should measure the contribution of the "magnetic" term to approximately 10% at $Q^2 \approx 0.1\ GeV^2$, including an estimated uncertainty in the "axial vector" term.

We now turn to the main point of this paper, namely the feasibility of extracting $G^{(S)}_E(q^2)$ from a measurement of $G^Z_E(q^2)$ at finite q^2. Examination of Eqn. 1 shows that, for fixed q^2, one would like ε near unity to enhance the "electric" contribution. This means that the scattering angle must be small so the beam energy E must be relatively large. Note that an additional advantage of $\varepsilon \approx 1$ is that the "axial vector" term is suppressed.

How large a contribution might we expect $G^{(S)}_E(q^2)$ to make to $G^Z_E(q^2)$? At first glance, Eqn. 2 would suggest that the contribution is small since $G^{(S)}_E(q^2)$ (as well as $G_{E_n}(q^2)$) must be zero at $q^2 = 0$ while $G_{E_p}(0)$ equals unity. However, since $\sin^2\theta_W \approx 0.23$, the contribution of G_{E_p} is suppressed by more than an order of magnitude. Also, it is reasonable to expect that $G^{(S)}_E(q^2)$ is comparable in magntitude to $G_{E_n}(q^2)$. At low Q^2 we expect[11] $F^{(S)}_1(q^2) = \frac{1}{6}r_s^2Q^2$ where r_s^2 is the "strangeness radius" of the proton. For $r_s^2 = 0.15\ fm^2$ (which is consistent with the analysis of Ref.[11]) this gives $G_E^{(S)}(q^2) \sim F_1^{(S)}(q^2) = 2.3\tau$ whereas $G_{E_n}(q^2) \sim 2.5\tau$.[15] (Precise measurements of $G_{E_n}(q^2)$ should be forthcoming in the relatively near future.[16]) Finally, we note that since both the "electric" and "magnetic" terms in Eqn. 1 are proportional to τ (at low Q^2), their overall contributions to the asymmetry are also comparable. These statements are consistent with the observation by Beck[8] that a proposed measurement of $G_{E_n}(q^2)$ using this approach[13] becomes difficult to interpret given a plausible model of strange vector form factors.

The differential cross section for elastic ep scattering, integrated over 2π azimuthal angle, can be written in the laboratory system as[13]

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2\cos^2(\theta/2)}{Q^4}\left(\frac{E'}{E}\right)^2\frac{1}{1-Q^2/2ME}\frac{\varepsilon G_{E_p}^2(q^2)+\tau G_{M_p}^2(q^2)}{\varepsilon(1+\tau)} \tag{3}$$

Here $Q^2 = 4EE'\sin^2(\theta/2)$ where E (E') is the incident (scattered) electron energy, and $E' = E/[1+(2E/M)\sin^2(\theta/2)]$. At forward scattering angles (i.e. θ small) the cross section is relatively insensitive to E (for fixed Q^2) since $E' \approx E$ and $Q^2 << 2ME$. It is therefore only important to make sure the energy is high enough so that the $\varepsilon \approx 1$ for the desired Q^2. Note that with a fixed beam energy, events can in principle be separated over a rather wide range of Q^2. This might be done with some large aperture spectrometer for the scattered electrons operating at some forward angle, or perhaps by detecting the recoil protons with provision for measuring their energy.[17]

Since the asymmetry A from Eqn. 1 is very small ($\sim 10^{-6}$), it is important to check that sufficient statistical accuracy can be achieved in a reasonable period of time. In Fig. 1 we plot the asymmetry from Eqn. 1, scaled by $G_FQ^2/\pi\alpha\sqrt{2} = 3.6\times10^{-4}\,(Q^2/1\ GeV^2)$. We take $G_M^{(S)}(q^2)$ and $G_E^{(S)}(q^2)$ from Ref.[11] using "Fit 7.1" from their Table 1. We use the standard dipole form for $G_A^Z(q^2)$[8]

and set the strange axial vector contribution to zero. We show the statistical accuracy one would achieve under certain assumptions by plotting points with error bars on top of the calculated curve. We have assumed $E = 2\ GeV$, a luminosity $\mathcal{L} = 1.3 \times 10^{38}/cm^2 \cdot sec$ (i.e. 50 μA on a 10 cm long liquid hydrogen target), a 50% polarized electron beam, 300 hr of data taking, and six equally spaced Q^2 bins from 0.1 GeV^2 to 0.3 GeV^2. We also show the calculated curves assuming that $G_E^{(S)}(q^2) = 0$, $G_M^{(S)}(q^2) = G_E^{(S)}(q^2) = 0$, and $G_A^Z(q^2) = G_M^{(S)}(q^2) = G_E^{(S)}(q^2) = 0$.

A number of points are clear from Fig. 1. First, the effect of $G_M^{(S)}(q^2)$ and $G_E^{(S)}(q^2)$ (as calculated from Ref.[11]) is both measurably large, and much larger than the effect from $G_A^Z(q^2)$ which is itself already known to $\sim 30\%$ including a possible strange axial vector contribution. Second, the effect of $G_M^{(S)}(q^2)$ is comparable to that from $G_E^{(S)}(q^2)$, and its normalization at $Q^2 \approx 0.1\ GeV^2$ should be known in the near future.[10] Until a backward angle measurement is done at higher Q^2, it will be necessary to assume some q^2 dependence of $G_M^{(S)}(q^2)$ to thoroughly interpret this experiment. Finally, the statistical precision shown in Fig. 1 is sufficient to establish an effect beyond the addition of $G_A^Z(q^2)$ and $G_M^{(S)}(q^2)$. The experimental asymmetry (including the beam polarization) is (in the worst case) $\sim 6 \times 10^{-7}$. This is 30 times larger than the ultimate systematic error achieved in a recently completed similar experiment.[18]

In conclusion, we have shown that it should be possible to measure the strangeness form factor $G_E^{(S)}(q^2)$ for the proton by using parity violation in elastic $\vec{e}p$ scattering at forward angles. The use of certain models, and hopefully other future experiments at backward angles, should allow one to extract the slope of the form factor $F_1^{(S)}(q^2)$ near $q^2 = 0$ and extract the strangeness radius of the proton.

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[1]J.Ashman, *et.al.*, Nucl.Phys. B328, 1 (1989); J.Ashman, *et.al.*, Phys.Lett. B 206, 364 (1988)

[2]John Ellis and Robert Jaffe, Phys.Rev.D 9, 1444 (1974)

[3]L.M.Sehgal, Phys.Rev.D 10, 1663 (1974)

[4]R.L.Jaffe and Aneesh Manohar, Nucl.Phys. B337, 509 (1990)

[5]R.L.Jaffe and C.L.Korpa, Comm.Nucl.Part.Phys. 17, 163 (1987)

[6]David B. Kaplan and Aneesh Manohar, Nucl.Phys. B310, 527 (1988)

[7]R.D.McKeown, Phys. Lett. B 219, 140 (1989). Note that in Eqn.7 the factor in front of the axial term should be $(E+E')/2M_N$, instead of $(E+E')/M_N$.

[8]D.H.Beck, Phys. Rev. D 39, 3248 (1989)

[9]R.Decker and Th.Lieze, Phys.Lett.B 246, 233 (1990)

[10]D.H.Beck and R.D.McKeown, *et.al.* (The SAMPLE Collaboration), MIT-Bates Experiment 89-06

[11]R.L.Jaffe, Phys.Lett. B 229, 275 (1989)

[12]R.N.Cahn and F.J.Gilman, Phys. Rev. D 17, 1313 (1978)

[13]T.W.Donnelly, J.Dubach, and Ingo Sick, Phys. Rev. C 37, 2320 (1988)

[14]M.J.Musolf and Barry R.Holstein, Phys.Lett.B 242, 461 (1990)

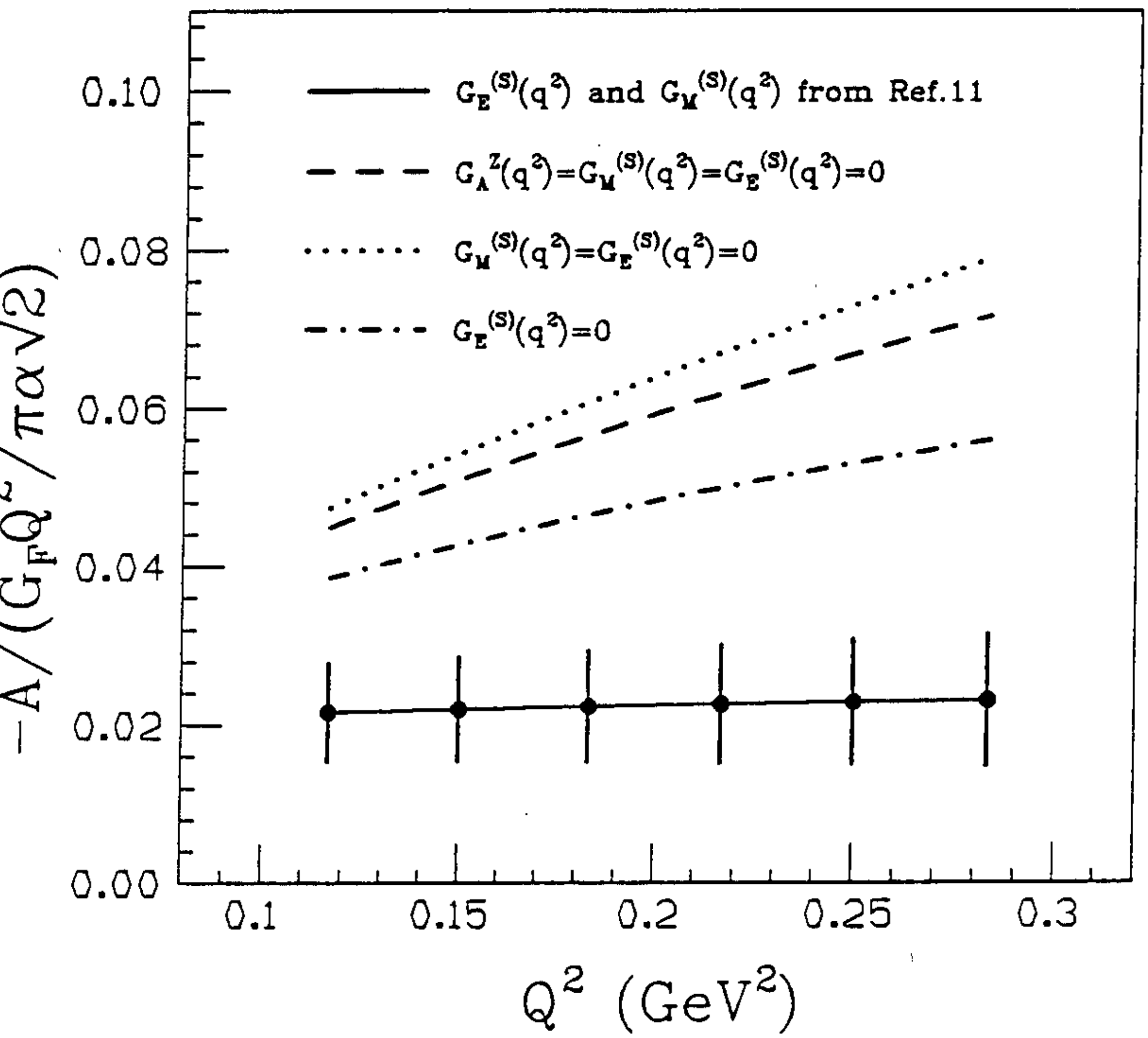
[15]S.Platchkov, *et.al.* Nucl.Phys. A510, 740 (1990)

[16]S.Kowalski and R.Madey, *et.al.*, MIT-Bates Experiment 85-05

[17]J.Napolitano, in "Parity Violation in Electron Scattering", workshop at California Institute of Technology, R.D.McKeown and E.J.Beise, *ed.*, World Scientific (1990)

[18]P.A.Souder, *et.al.*, Phys.Rev.Lett. 65, 694 (1990)

FIG. 1. Parity violating asymmetry as a function of Q^2 for a beam energy $E = 2\ GeV$. The asymmetry is calculated using Eqn. 1 and Eqn. 2 and is divided by the factor $G_F Q^2/\pi\alpha\sqrt{2} = 3.6 \times 10^{-4}\,(Q^2/1\ GeV^2)$. The strange-nucleon form factors are calculated using Ref.[11]. The error bars represent the statistical precision one would achieve with a 50% beam polarization, a luminosity of $1.3 \times 10^{38}/cm^2 \cdot sec$, and 300 hr of data taking using a detector covering the full azimuthal angle. The effect of the axial vector and strange form factors on the result are also shown.



$G_E^{(S)}(q^2)$ and $G_M^{(S)}(q^2)$ from Ref.11
$G_A^Z(q^2)=G_M^{(S)}(q^2)=G_E^{(S)}(q^2)=0$
$G_M^{(S)}(q^2)=G_E^{(S)}(q^2)=0$
$G_E^{(S)}(q^2)=0$
$-A/(G_F Q^2/\pi\alpha\sqrt{2})$
0.00
0.02
0.04
0.06
0.08
0.10
0.1
0.15
0.2
0.25
0.3
Q^2 (GeV^2)